GR Reference Sheet:
Kerr Black Hole

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Boyer–Lindquist Coordinates

The spacetime metric is

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4aMr\sin^2\theta}{\Sigma} dt\, d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma\, d\theta^2 + \left(r^2 + a^2 + \frac{2Mra^2\sin^2\theta}{\Sigma}\right) d\phi^2 \tag{1}$$

where a is the black hole spin and

$$\Sigma \equiv r^2 + a^2\cos^2\theta, \tag{2}$$

$$\Delta \equiv r^2 - 2Mr + a^2. \tag{3}$$

The volume element is

$$\sqrt{-g} = \Sigma\sin\theta \tag{4}$$

The lapse function and shift vector are

$$\alpha = \left(\frac{\Sigma\Delta}{2Mr(r^2+a^2) + \Sigma\Delta}\right)^{1/2} \tag{5}$$

$$\beta^r = \beta^\theta = 0\ , \quad \beta^\phi = -\frac{2aMr}{(r^2+a^2)\Sigma + 2a^2Mr\sin^2\theta} \tag{6}$$

Extrinsic curvature:

Geodesic equations: